THE ROLE OF STATISTICIAN IN AGRICULTURAL AND ANIMAL HUSBANDRY RESEARCH*

Dr. P. C. Raheja (I.A.R.I., New Delhi) opening the Symposium said that the statisticians have played an important role in systematising the technique of experiments in the field and in the laboratory and in enabling the scientists to correctly evaluate their results. No agronomical trial is now considered satisfactory unless its layout is based on the three basic principles of replication, randomisation and local control. The implications of these principles are also being exploited on a wider scale in experiments on cultivators' fields in which the replications are spread over several villages. The ingenious device of confounding enables us to increase the precision of experiments involving several factors tried at different levels.

Statisticians have played an equally important role in the realm of plant breeding and genetics. It is now possible to test segregation ratios and to determine efficiently the linkage values, and identify the linked characters in inheritance. Animal genetics, eugenics and bio-assay of insecticides are based upon the most modern procedures in statistics.

Another field in which the statisticians have played a very significant role is that of the surveys. Well-known examples in the field of agriculture are the surveys for the estimation of area and yield of crops, farm income, labour employment, etc. The technique has also proved useful in the assignment of improvements introduced by such measures as improved varieties of crops, irrigation, fertilizers, etc.

Basic mathematical research has led to the discovery of efficient statistical tests like x^2, t, and z or F by K. Pearson, 'student' and Fisher respectively. These have enabled non-mathematical agricultural workers in the sciences of agronomy, biology, physics and chemistry to evaluate their results at their true worth.

While commending all this development the speaker depricated the orthodoxy of statisticians and remarked that some of the outstanding discoveries in the field of the biological sciences have originated from keen observations and several of them passed the statistical test much

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* Symposium held during the Ninth Annual Meeting of the Indian Society of Agricultural Statistics. Dr. R. J. Kalamkar, Additional Agricultural Commissioner, Indian Council of Agricultural Reseach, presided on the occasion.

later. As an illustration he cited the introduction of the important sugarcane varieties like Co. 205, Co. 210, Co. 213 and Co. 214 which laid the foundations of the sugar industry in the country, and the new Pusa wheats such as NP 4, NP 52, NP 710 and NP 718 which have spread over practically the major portion of wheat-growing tracts. He requested the statisticians not to lay too much emphasis on the use of the very orthodox methods of statistical analysis in agricultural and allied experimentation. He cited the example of the wheat variety NP 718 which although not proved to be significantly superior to the variety C 591 in trials for five years at the Indian Agricultural Research Institute is now replacing the latter variety rapidly not only in Delhi but in the major portion of wheat growing tracts, because it gives $1\frac{1}{2}$ to 3 maunds more yield per acre than C 591 in the cultivators' fields.

He appreciated the two-way traffic of ideas flowing between scientific workers and statisticians, a process benefiting both the sciences immensely. The danger, however, arises when the research worker begins to assume from his limited knowledge that he had become a statistician and tries to draw conclusions which cannot stand the scrutiny of scientific tests. Similarly the statisticians would do well to act as advisers rather than assume the role of leaders of other agricultural sciences. They are playing a very vital role in the planning of experimental work, but it would be better to leave the formulation of the research programme and reporting to the workers in the respective fields. This will encourage scientific workers to seek their co-operation and this would prove mutually advantageous.

In conclusion, Dr. Raheja said that science is not science unless it is measurable and interpretable mathematically. Mathematics is the life and soul of exactness. Logically there is a very strong need for mutual co-operation between the statisticians and agricultural workers and this has to be the corner-stone for progress in agricultural research in the years to come.

Speaking on the role of the statistician in the field of animal husbandry research and development Shri K. P. R. Kartha (I.C.A.R., New Delhi) said that animal production is extremely complicated and is influenced by a multiplicity of factors connected with heredity and environment. Many of these factors are measurable and lend themselves to statistical treatment. Consequently, statistical studies are essential for the precise planning of projects, review of progress and the final evaluation of results.

The application of statistical methods to animal husbandry data is comparatively recent origin. Even so, as a result of the establishment of the I.C.A.R., considerable progress has been achieved in India in the fields of animal experimentation and scientific analysis of other animal husbandry data. At present every scheme of research in animal husbandry is designed in consultation with the statistician and no technical progress is adopted without his full approval. No conclusion is accepted as final unless it is statistically valid.

The statistician has a much wider role in the field of animal husbandry development as his active co-operation with the animal husbandry worker can result not only in reducing waste of effort and accelerating the pace of progress, but also in the development of the basic sciences of breeding, nutrition and protection against disease.

Some work has already been done with accumulated breeding data obtained at Bangalore, Hosur and Etah. However, the wider application of statistical methods in India is considerably handicapped by the paucity of recorded data. In regard to cattle, data are available only in about 150 farms, which together do not maintain more than about 15,000 out of the country's population of 70 million cows and buffaloes. Even these are not suitable for precise statistical study because of the large differences in feeding, management, etc. In the case of sheep, poultry and other livestock the data available are still less. Large-scale data from rural areas cannot be had at the present stage of our development. In this respect the co-operation of governments is needed to see that adequate funds and facilities are made available for the maintenance of detailed and authentic records.

Meanwhile, to the extent to which data permit, there are several items on the study of which statistical skill can be employed profitably. Examples are provided by the complex phenomena of animal behaviour and animal response to treatment. The true measure of the milking capacity of a cow needs to be defined. As a corollary to this, an attempt has to be made to evolve a standardised lactation curve which takes account of the differences in age, order of lactation, dry periods, calving interval and season of calving. An intensive study of the lactation curve would be of very great use and significance. It is desirable to know whether persistency is inherited and what relationship the maximum milk yield and that at various stages of lactation bears to the total milk yield in the lactation. In arranging for the recording of milk production in the villages it would be useful to be able to estimate the total production of the animal by taking records at intervals instead of recording daily yields. The wider this interval the less would be the cost of milk recording.

Sampling techniques open up vast possibilities in the field of animal husbandry. These have been found useful in investigations on the cost of production of milk, in checking up the accuracy of census data, in judging the potentialities of milk production under better feeding and management, in estimating the average production of milk in India, and in sampling for measurement of wool quality.

Shri Kartha said that so far no yardstick is available for measuring the working capacity of bullocks. However, certain investigations have been conducted at Allahabad and Izatnagar and he felt that with the help of the statistician it would be possible to arrive at a satisfactory measure.

Progeny testing is universally recognised to be the only sound basis for selection of breeding stock, and a number of indices representing transmitting ability of cattle have been worked out in foreign countries. With the extension of our own research and development, enough data may be available to evolve an index which is suitable under Indian conditions.

The services of the statistician are essential in many other instances like digestibility trials, nutritive values of feeds, rate of growth of calves and its relationship with work or milk production, methods of breeding, the efficacy of biological products, the effect of diseases on production, study of immunity and several matters relating to the economics of production.

The speaker finally remarked that the effectiveness of statistical applications to the examination of animal husbandry data as well as its proper recording could be considerably increased if the worker himself gets at least an elementary training in statistical methods, and concluded with the warning that the statistician also should have a sound knowledge of the practices in animal husbandry and the circumstances under which data are collected and compiled, so as to guard himself against pitfalls and misleading interpretations.

Dr. S. S. Prabhu (I.V.R.I., Izatnagar) explained the varied applications that statistics has in animal husbandry with the help of examples taken from his own special branch of study. The first example related to methods for estimating semen-producing capacity of a bull. He said that this capacity could be correctly determined by means of an " exhaustion test ", but the method is time consuming and becomes impracticable for routine use. Correlation studies, however, established that examination of first, or average of first-two ejaculations gave a very fair estimate of the semen-producing capacity of the bull.

The second problem was on sampling for estimating the sperm density or concentration in an ejaculate. The general technique is to sample the original ejaculate at four stages: (i) taking the original 0·1 c.c. for first dilution; (ii) taking 1 c.c. for the second dilution; (iii) taking a drop for charging the hæmacytometer; and (iv) examining the squares of the hæmacytometer. Carefully planned experiments, however, showed that a better estimate of the sperm density could be had by taking two samples at each of the first three stages and examining only half the total number of squares.

As a third example he described an experiment to determine the effect of levels of protein in rations of bulls on semen quality. In this case the statistical treatment of the data showed no marked effect of the protein levels, but a close scrutiny of the basic data revealed many important facts.

Dr. Prabhu concluded by depricating the blind application of statistical methodology without a clear understanding of the problem itself and the limitations imposed by the unpredictable behaviour of the experimental material such as large animals.

PROFESSOR JAMES WARNER (Agricultural Institute, Allahabad) gave a number of instances where statisticians could be of great help to research workers in animal husbandry and agriculture. He began with a description of the several co-operative milk supply unions being established in the country with Government help. These unions are intended to increase the supply of milk to towns and to give the producer of milk a better price. The attraction of a better price for milk may, however, result in the producer consuming lesser amounts in his home. Since, from a national point of view, an increase in the consumption of milk in the village homes is fully as important as an increase in the consumption of milk in our cities, studies should be made to ascertain whether and if so, just how, the establishment of milk supply unions affects the consumption of milk by the producer and members of his family. Statisticians could help in planning and carrying out such studies.

He next referred to the many key village blocks and artificial insemination schemes which are visualised to cover the entire country by the end of the Second Five-Year Plan. Artificial insemination has one great disadvantage in that man becomes responsible for detecting œstrus in the cow and for assuring insemination at that time. As the œstral period is of short duration and often very faint, man cannot detect it so successfully as can the male of the species. This is also borne out by the fact that the incidence of conception per insemination by artificial means has seldom been found greater than that by natural means, and even when it has been so found, the difference in favour of artificial insemination is rarely phenomenal. In these circumstances there is a danger that the average age at first calving and the average calving interval of our cows might actually increase. Studies to ascertain the extent to which age at first calving and calving interval of our cows might change with the use of artificial insemination should be made. Such studies require careful planning, execution and interpretation and the statistician must help at all three stages. This work should be done soon and the considerable economic harm which might occur with artificial insemination should be avoided.

Prof. Warner related an example given by Dr. J. L. Lush where the experimenter was trying to increase litter size in pigs by breeding only sows born in large litters. When he failed in his attempt, a careful study revealed that he had, at the time of selecting individual sows for breeding, unconsciously selected the wrong sow rather often. It is interesting to examine how often we violate the basic objective of our animal and crop experiments by such unconscious errors. Statisticians may indeed be able to help research workers in avoiding such errors.

The speaker described how cows which freshen in the autumn gave more milk than cows which freshen in the spring in the U.S.A. This was due to the fact that autumn freshening allowed the high production effect of early lactation to occur at a different time from that of the high production effect of fresh spring grass and fine weather, whereas in spring freshening both these effects occurred simultaneously followed by the low production effect of late lactation and dry short grass in the summer months. In India too production of milk goes down rapidly in April, May and June and remains low until September and October. Statisticians can help in setting up experiments, or in analysing data already available from recorded herds, which may show what is the best time for our cows to freshen, and therefore when it is best to breed them so as to assure maximising the high production effects of stage of lactation, feed and weather.

Professor Warner concluded with the remarks that additional knowledge of the science of statistics and of the art of its application in the planning, execution and analysis of animal husbandry and agricultural experiments could be profitably given to all workers in these fields of study. One excellent way to assure this for future workers is to include more course in statistics in our animal husbandry and agricultural college courses. This is urged as being fundamental to the wider appreciation of the utility of statistics and of the statistician and to the economy of experimentation.

Dr. K. Kishen (Department of Agriculture, U.P., Lucknow) stressed the increasingly important role being played by statisticians in agricultural and animal husbandry research in this country since the first publication of R. A. Fisher's well-known book *Statistical Methods for Research Workers,* in 1925 and the noteworthy efforts of the Indian Council of Agricultural Research in making available to Indian workers the modern statistical techniques and procedures introduced by him. He further pointed out that lack of suitably qualified statisticians was one of the principal reasons for the unsatisfactory progress made by underdeveloped countries in the field of agricultural and animal husbandry research. As an instance he gave the example of Afghanistan which he had visited recently, where there were no competent agricultural statisticians and in consequence the experimental resources of the country were not being utilised properly. He gave an illustration of the nature of defects in the layouts used there and commended the use of exploratory factorial experiments in order to ascertain which factors could be omitted in future and which could be retained for further trial. It is only through such an evolutionary system of experimentation that the optimum utilisation of the experimental resources of a country can be ensured.

Dr. Kishen remarked that the statistician has a dynamic role to play at all the stages of an experimental investigation, namely, in its planning, its execution and finally in the analysis and interpretation of its data. Although the agricultural and animal husbandry research workers may be familiar with the layout plans commonly adopted in their investigations, it is necessary to emphasize that the advice of an expert statistician will often result in more appropriate and efficient layout plans. Also in laying out complete designs they would often be greatly benefited by his advice since the analysis of these designs is difficult particularly where there are missing plots. It is, therefore, desirable for research workers to get their layout plans tested by an expert statistician before actually carrying out an experiment.

The role of the statistician at the time of the actual execution of the plan is no less important. It is for him to ensure that as uniform and homogeneous a material as can be made available is selected for experimentation. It is also one of his important functions to see that no avoidable damage occurs to the experiment which it is being carried out, as any damage to the experiment cannot possibly be repaired by any statistical expedient. Finally, the most important part which the statistician plays is in the statistical analysis and interpretation of the data collected. The research workers are often apt to perform only routine statistical analysis of the data and content themselves with only testing the significance of the results at the conventional 5% and 1% levels of probability. No importance is usually attached by them to results which fail to reach these conventional levels of significance. Such a superficial approach in the analysis and interpretation of data collected at considerable cost is to be deprecated as it is important to obtain the maximum information from these data for future guidance, which only a competent statistician can be safely entrusted to do.

Dr. Kishen concluded by emphasizing that a statistician is only a collaborator of other research workers and must work in close harmony with them if the best results are to be achieved.

Dr. V. G. Panse (I.C.A.R., New Delhi) said that some 20 years back, the statistician was regarded as a handy assistant to the agricultural research officer and the results of statistical analysis were used by the latter only if they turned out to be in accordance with his expectations but ignored otherwise. The statistician of today has, however, established himself as an unavoidable appendage to agricultural research, and it is being generally conceded that unless the design of an experiment or a sample survey is on statistically sound lines and is approved by the statistician, the results would not be accepted as conclusive. The position is, however, not very satisfactory as the statistician is usually told to give a plan to fit a particular set of treatments and the experimenter still reserves the right to draw his own conclusions which he considers to be practically sound from the data. It is thus not possible to make full use of the statistical science for promoting research on efficient and economic lines.

Dr. Panse said that research can progress and produce results even without the application of statistics, but with the help of this science a good deal of waste of time and resources in arriving at the same results can be saved, thereby making for a more effective utilisation of the limited resources available for research. The statistician can contribute to this by helping the research worker through discussion to arrive at an optimum set of treatments, which while meeting the objectives that the latter has in view, utilises more completely his experimental resources in providing information. He illustrated this point by an example given by Finney where instead of conducting two experiments, each on one factor with 16 plots, the 32 plots available could be utilized far more efficiently by including in a single experiment as many as 5 factors without any loss of information on any factor but adding extra information on their mutual interaction. At the stage of interpretation also, there need not be any conflict between the so-called practical conclusions and statistical inferences. If there is any contradiction between the two, either the one or the other is wrong.

He said that in the interest of greater efficiency and economy of research, the statistician's specialization in quantitative thinking ought really to be used on a wider front than only for designing or interpreting isolated experiments or other investigations. In deciding upon research policy and programme of an experimental station or of a State department of agriculture the problem is essentially one of optimum allocation of the resources among different branches of research and among different lines of investigation. If research is not an end in itself, but is undertaken for securing results for practical application, then allocation of resources for producing the best results has to be based on what is essentially a statistical analysis of past results and knowledge. It is not suggested that the statistician is immediately in a position to carry out this analysis. Some of the essential data may be lacking, some technical methods for utilizing past information for formulation of research policies would also have to be developed.

He emphasized in this connection the value of associating a statistician in formulating plans and policies for research, whether of an experimental station, a State department or the country as a whole so that his quantitative view is brought to bear on decisions that may be taken.

Dr. Panse further said that the agricultural statistician could make a much greater contribution to research, if certain limitations could be eliminated. One of the first needs is to provide competent statistical services at experimental stations, teaching institutions, agriculture and animal husbandry departments, etc. The second is to provide for a better training in statistics of the research worker himself. It would be ideal if the research worker could act as his own statistician. Although this is not always possible, he should at least be trained to understand the basic principles and the fundamental approach of statistical methods, so that he is able to appreciate the statistician's view-point and is able to accept his advice with greater sympathy and confidence. The statistician, on the other hand, must realise that he cannot work in abstract, but must acquire a sufficient first-hand knowledge of the technology of the research field, in which he is to tender advice. He should, in fact, frequently participate in the conduct of the experiment or investigation, which he has designed or for which he has laid down some operational procedure. It is only then that he will be able to meet the needs and command respect from his research colleagues. In conclusion Dr. Panse said that the statistician must not be regarded either as a handmaid of other scientists or their master. His real position is that of a collaborator and he must work in harmony with other research workers without any inferiority or superiority complex. That statistics properly applied can help research immensely has been amply demonstrated. To exploit its great potentialities research workers and statisticians must develop the spirit of working with mutual respect and understanding.

The Chairman, DR. R. J. KALAMKAR, Additional Agricultural Commissioner, Indian Council of Agricultural Research, in his closing remarks pointed out that there was a time when plant breeding was regarded as an art and its success depended largely upon the personal observation of the plant breeder. It has acquired now the status of a definite science mainly through the use of modern statistical techniques. The factorial concept of planning experiments has replaced the single factor investigations of the past, thus contributing greatly to the comprehensiveness and usefulness of the experimental programmes in the field of agriculture and animal husbandry. He went on to emphasize that for making the best use of the knowledge of the Statistician, it was essential that he should be associated with the research workers from the planning stage of the experiment and not merely called upon to salvage information from the data of experiments in the planning of which he had no hand. While the Statisticians are playing an important role in the fields of agriculture and animal husbandry, it was incumbent upon them to acquire a good knowledge of the various subject-matter fields in order to make an effective contribution in the solution of many intricate problems with which the present-day research workers in the field of agriculture and animal husbandry are concerned.

ESTIMATION OF COMPONENTS OF VARIANCE AND OF HERITABILITY WITH THEIR STANDARD ERRORS

By A. V. K. Sastri

Indian Council of Agricultural Research

1. Introduction

The estimation of components of variation has wide applications in genetic research. The plant or animal breeder engaged in the improvement of his material through selection is primarily interested in knowing the amount of variation that is due to additive effects of the genes, dominance effects of the genes and due to environmental causes, for the effectiveness of his selection depends on the amount of genetic variation present in his stock. Methods of estimation of the fraction of genetic variation to the total phenotypic variation, *i.e.*, coefficient of heritability, have been studied previously by Panse and Bokil (1948) in plants and by Lush (1940) in animals. Heritability of a particular character in animals is generally estimated by the intra-sire regression method, being twice the coefficient of the intra-sire regression of daughters' records on dams' records. It is clear that application of this method requires information on the dams. Methods of estimation of heritability when the dams' records are not known are, however, available. These methods relate to analysis of sib data and are dealt with by several workers. The errors with which the estimates of components of variation are obtained, are, however, not given. It is the object of this paper to present the method of estimation of components of variation and hence heritability with their standard errors, when the progeny records only are available. The method has been illustrated with the records of the first clip wool yield of 510 ewes belonging to the first generation, under the sheep breeding scheme, Hissar, distributed over the period 1937–38 to 1946–47. The mathematical procedure of estimation is described below.

2. Method of Estimation

The phenotypic expression of a character can be considered as the sum of a genotypic effect and an effect attributable to environment. Symbolically $x = y + e$, where x is the phenotypic value, y the genotypic value and e the environmental deviation. If the contribution made by non-heritable agents is assumed independent of genotypic effect, the phenotypic variance is $\sigma_p^2 = \sigma_y^2 + \sigma_e^2$, where σ_y^2 is the variance of genotypic effects and σ_e^2 is the variation resulting from non-heritable causes. The method of estimation of genotypic and environmental variances is as follows:—

2.1. Estimation of Components of Variance and Co-variance

The genotypic variance σ_y^2 in a character can be considered as composed of the following three components:

1. Additive genetic variance σ_g^2
2. Variance due to dominance deviations from the additive scheme σ_d^2
3. Variance due to epistatic deviations from the additive scheme.

If a character is governed by genes at n loci and if there is no epistasis, $y = y_1 + y_2 + \cdot + y_i + \cdot + y_n$, where y_i is the effect of the gene at the ith locus ($i = 1, 2, \ldots n$). If, in addition, there is no linkage among the genes at different loci, $\sigma_g^2 = \sum_{i=1}^{n} \sigma_{g_i}^2$, $\sigma_d^2 = \sum_{i=1}^{n} \sigma_{d_i}^2$ and $\sigma_y^2 = \sum_{i=1}^{n} \sigma_{y_i}^2 = \sigma_g^2 + \sigma_d^2$, where $\sigma_{g_i}^2$ is the additive genetic variance and $\sigma_{d_i}^2$ the variance due to dominance deviations resulting from segregation at the ith locus. Considering that either of two allelomorphs A and a of a gene may occupy a given locus i, the genotype of a diploid organism with respect to that locus may be AA, Aa or aa. In a population mating at random, if the alleles A and a of a gene occur with frequencies q and $1 - q$ respectively, the three genotypes AA, Aa and aa occur with the average frequencies q^2, $2q(1-q)$, $(1-q)^2$ in each generation. Let the average effects of the three genotypes on the magnitude of the character under consideration be d, h and $-d$ respectively. The additive genetic variance $\sigma_{g_i}^2$ is estimated by the sum of squares due to regression of average effect of the genotypes on the number of favourable alleles. This can be shown to be equal to $2q(1-q)[d + h(1-2q)]^2$. The genotypic variance $\sigma_{y_i}^2$ works out to be $2q(1-q)[d + h(1-2q)]^2 + 4h^2q^2(1-q)^2$. Hence $\sigma_{d_i}^2$ is equal to $4h^2q^2(1-q)^2$. Now, summing over all the genes, when there is no epistacy or linkage, we have $\sigma_g^2 = 2\Sigma q(1-q)[d + h(1-2q)]^2$, $\sigma_d^2 = 4\Sigma h^2q^2(1-q)^2$. These are respectively equal to $\frac{1}{2}D$ and $\frac{1}{4}H$ in Mather's (1949) notation. The average value, with respect to that character, of the progenies produced when a male with genotype AA is mated with the females having the genotypes AA, Aa and aa will be d, $(d + h)/2$ and h respectively. Taking all the types of parent in turn, we can form a table as given below showing the frequencies of various types of matings and the average values of the progenies.

TABLE I

Frequencies of the different genotypes and the average values of the progenies produced by different types of matings

Mating Male	Female Frequency	AA q^2	Aa $2q(1-q)$	aa $(1-q)^2$
		Average value of the progeny		
AA	q^2	d	$\frac{d+h}{2}$	h
Aa	$2q(1-q)$	$\frac{d+h}{2}$	$\frac{h}{2}$	$\frac{h-d}{2}$
aa	$(1-q)^2$	h	$\frac{h-d}{2}$	$-d$

From the above table, the average values of the progeny for the three types of males can be shown to be the following:—

TABLE II

Progeny mean of the three types of males

Genotype of Male	Frequency	Progeny mean	General mean
AA	q^2	$qd + (1-q)h$	$(2q-1)d$
Aa	$2q(1-q)$	$\frac{h}{2} + (2q-1)\frac{d}{2}$	$+ 2q(1-q)h$
aa	$(1-q)^2$	$qh - (1-q)d$	

From Table II, it can be seen that the expected total genetic variance among paternal half-sib families $= \frac{1}{8} D = \frac{1}{4} \sigma_g^2$. From Table I, it can be found that the expected total genetic variance of means of progenies from different females but the same male, *i.e.*, between full-sib families within half-sib family groups is equal to $\frac{1}{8} D + \frac{1}{16} H$ or $\frac{1}{4} \sigma_y^2$ and the expected total mean genetic variance within the full-sibs $= \frac{1}{4} D + \frac{3}{16} H = \frac{1}{2} \sigma_g^2 + \frac{3}{4} \sigma_d^2 = \frac{1}{2} \sigma_y^2 + \frac{1}{4} \sigma_d^2$.

Now, consider s sires belonging to a population, the ith sire having been mated to d_i dams belonging to the same population and having in all N_i daughters. Let the jth dam of the ith sire have n_{ij} daughters so that $\sum_{j=1}^{d_i} n_{ij} = N_i$. Let N denote the total number of daughters from all the s sires so that N is equal to $\sum_{i=1}^{s} N_i$. If y_{ijk} denotes the value of the character observed in the kth daughter of jth dam of ith sire, let $y_{ijk} = \mu + s_i + d_{ij} + e_{ijk}$, where μ is the effect common to all the individuals, s_i a deviation common to all the progeny of ith sire, d_{ij} a deviation common to the progeny of jth dam and ith sire and e_{ijk} a random deviation. Let the contributing effects e_{ijk}, d_{ij} and s_i be independently distributed with mean zero and variances σ^2, σ_m^2 and σ_s^2 respectively. $i = 1, 2, \ldots, s$; $j = 1, 2, \ldots, d_i$; $k = 1, 2, \ldots, n_{ij}$.

Let

$$\bar{y}_{ij} = \frac{\sum_k y_{ijk}}{n_{ij}}$$

$$\bar{y}_i = \frac{\sum_j \sum_k y_{ijk}}{N_i}$$

$$\bar{y} = \frac{\sum_i \sum_j \sum_k y_{ijk}}{N} = \sum_i \frac{N_i \bar{y}_i}{N}$$

Then the variation among N daughters can be partitioned as indicated in Table III.

TABLE III

Analysis of variance of N *daughters*

Source of Variance	D.F.	S.S.	M.S.	Expected value of M.S.
Between sires ..	$s-1$	$\sum_i N_i (\bar{y}_i - \bar{y})^2$	A	$\sigma^2 + \lambda_2 \sigma_m^2 + \lambda_3 \sigma_s^2$
Between dams within sires	$\sum_i (d_i - 1)$	$\sum_i \sum_j n_{ij} (\bar{y}_{ij} - \bar{y}_i)^2$	B	$\sigma^2 + \lambda_1 \sigma_m^2$
Within dams within sires	$\sum_i \sum_j (n_{ij} - 1)$	$\sum_i \sum_j \sum_k (y_{ijk} - \bar{y}_{ij})^2$	C	σ^2
Total	$N-1$	$\sum_i \sum_j \sum_k (y_{ijk} - \bar{y})^2$		

where

σ^2 is the sum of the variance attributable to environmental causes and the genetic variance within the full-sibs,

σ_m^2 is the variance of dam effects,

σ_s^2 is the variance of sire effects,

and

$$\lambda_1 = \frac{1}{\sum_i (d_i - 1)} \left[N - \sum_i \left(\frac{\sum_j n_{ij}^2}{N_i} \right) \right]$$

$$\lambda_2 = \frac{1}{s-1} \left[\sum_i \left(\frac{\sum_j n_{ij}^2}{N_i} \right) - \sum_i \sum_j \frac{n_{ij}^2}{N} \right]$$

$$\lambda_3 = \frac{1}{s-1} \left[N - \sum_i \frac{N_i^2}{N} \right].$$

From Table III, the estimates of the components of variance are given below:

Variance Component	Estimate
σ^2	C
σ_m^2	$\frac{B-C}{\lambda_1}$
σ_s^2	$\left[A - C - \lambda_2 \frac{B-C}{\lambda_1} \right] \frac{1}{\lambda_3}$

If there is no maternal influence, σ_s^2 and σ_m^2 will contain only the genetic variance between the male progeny means and the genetic variance of means of progenies from different females but the same male respectively.

Thus we have

$$\begin{aligned} \tfrac{1}{4} D + \tfrac{3}{16} H + E &= C \\ \tfrac{1}{8} D + \tfrac{1}{16} H &= \frac{B-C}{\lambda_1} \qquad (1) \\ \tfrac{1}{8} D &= \left[A - C - \lambda_2 \frac{B-C}{\lambda_1} \right] \frac{1}{\lambda_3}. \end{aligned}$$

By solving these equations, we can obtain the estimates of D, H and E, where D and H are as defined previously and E is the environmental variance of the character.

2.2. Estimation of Standard Errors of Variance Components Estimates, and of Heritability

By solving the set of three equations in (1), we have

$$\hat{\sigma}_g^2 = \tfrac{1}{2}\hat{D} = \frac{4}{\lambda_1\lambda_3}\left[\lambda_1 A - \lambda_2 B + (\lambda_2 - \lambda_1)C\right]$$

$$\hat{\sigma}_d^2 = \tfrac{1}{4}\hat{H} = \frac{4}{\lambda_1\lambda_3}\left[-\lambda_1 A + (\lambda_2 + \lambda_3)B - (\lambda_2 + \lambda_3 - \lambda_1)C\right]$$

$$\hat{E} = \frac{1}{\lambda_1\lambda_3}\left[\lambda_1 A - (\lambda_2 + 3\lambda_3)B - (\lambda_1 - \lambda_2 - 3\lambda_3 - \lambda_1\lambda_3)C\right].$$

If σ_A^2, σ_B^2, σ_C^2 denote the variances of the mean squares A, B, C respectively in Table III, then since A, B, C are independent of each other,

$$S.E.(\hat{\sigma}_g^2) = \frac{4}{\lambda_1\lambda_3}\left[\lambda_1^2\sigma_A^2 + \lambda_2^2\sigma_B^2 + (\lambda_2 - \lambda_1)^2\sigma_C^2\right]^{\frac{1}{2}} \qquad (2)$$

$$S.E.(\hat{\sigma}_d^2) = \frac{4}{\lambda_1\lambda_3}\left[\lambda_1^2\sigma_A^2 + (\lambda_2 + \lambda_3)^2\sigma_B^2 + (\lambda_2 + \lambda_3 - \lambda_1)^2\sigma_C^2\right]^{\frac{1}{2}}$$

$$S.E.(\hat{E}) = \frac{1}{\lambda_1\lambda_3}\left[\lambda_1^2\sigma_A^2 + (\lambda_2 + 3\lambda_3)^2\sigma_B^2 + (\lambda_1 - \lambda_2 - 3\lambda_3 - \lambda_1\lambda_3)^2\sigma_C^2\right]^{\frac{1}{2}}.$$

The variances of the mean squares between and within classes source of variation in one-fold hierarchal classification under the assumptions of Model II of Eisenhart (1947), *i.e.*, when s_i, d_{ij} and e_{ij} are normally distributed, are given by Crump (1951). The results given by Crump could not, however, be applied in the present case as the variances of the mean squares between sires, between dams within sires and within dams within sires are required to get the standard errors of the estimates of components of phenotypic variation. Accordingly, the results were extended to the two-fold hierarchal classification and they turned out to be as follows:—

$$\sigma_A^2 = \frac{2\sigma^4}{(s-1)^2}\left[\frac{1}{N^2}\left(\sum_i \frac{N_i^2}{\omega_i}\right)^2 + \sum_i \frac{N_i^2}{\omega_i^2} - \frac{2}{N}\sum_i \frac{N_i^3}{\omega_i^2}\right]$$

$$= \frac{2\sigma^4}{\left[\sum_i (d_i - 1)\right]^2}\left[\sum_i \sum_j \frac{n_{ij}^2}{\omega_{ij}^2} + \sum_i \frac{1}{N_i^2}\left(\sum_j \frac{n_{ij}^2}{\omega_{ij}}\right)^2 - 2\sum_i \frac{1}{N_i}\sum_j \frac{n_{ij}^3}{\omega_{ij}^2}\right]$$

$$\sigma_C^2 = \frac{2\sigma^4}{\sum_i \sum_j (n_{ij} - 1)} \qquad \text{where } \omega_i = \frac{N_i \sigma^2}{\sigma^2 + N_i \sigma_s^2 + \frac{\sum_j n_{ij}^2}{N_i} \sigma_m^2}$$

$$\omega_{ij} = \frac{n_{ij}\sigma^2}{\sigma^2 + n_{ij}\sigma_m^2}$$

For calculation, the above expressions can be put in a convenient manner as shown below:

$$\sigma_A^2 = \frac{2\sigma^2}{s-1} [\sigma^2 + 2\lambda_3 \sigma_s^2 + 2\lambda_2 \sigma_m^2] + \frac{2}{(s-1)^2} [p_1 \sigma_m^4 + p_2 \sigma_s^4 + 2p_3 \sigma_m^2 \sigma_s^2]$$

$$\sigma_B^2 = \frac{2\sigma^2}{\sum_i (d_i - 1)} [\sigma^2 + 2\lambda_1 \sigma_m^2] + \frac{2p}{[\sum_i (d_i - 1)]^2} \sigma_m^4$$

where

$$p = \sum_i \sum_j n_{ij}^2 + \sum_i \left(\frac{\sum_j n_{ij}^2}{N_i} \right)^2 - 2 \sum_i \left(\frac{\sum_j n_{ij}^3}{N_i} \right)$$

$$p_1 = \sum_i \left(\frac{\sum_j n_{ij}^2}{N_i} \right)^2 + \frac{(\sum_i \sum_j n_{ij}^2)^2}{N^2} - \frac{2}{N} \sum_i \left\{ \frac{(\sum_j n_{ij}^2)^2}{N_i} \right\}$$

$$p_2 = \sum_i N_i^2 + \frac{(\sum_i N_i^2)^2}{N^2} - 2 \sum_i \left(\frac{N_i^3}{N} \right)$$

$$p_3 = \frac{\sum_i N_i^2 \sum_i \sum_j n_{ij}^2}{N^2} + \sum_i \sum_j n_{ij}^2 - 2 \sum_i \left(\frac{N_i \sum_j n_{ij}^2}{N} \right).$$

Approximate values of estimates of σ_A^2, σ_B^2 and σ_C^2 can be obtained by substituting the estimates of σ^2, σ_m^2 and σ_s^2 which can be obtained from the analysis of the variance Table III. Then the standard errors of $\hat{\sigma}_g^2$, $\hat{\sigma}_d^2$, $\hat{E}$ can be obtained by substituting the estimates of σ_A^2, σ_B^2 and σ_C^2 in the set of equations (2).

Now, by definition, the coefficient of heritability h^2 is give

$$\frac{\sigma_g^2}{\sigma_g^2 + \sigma_d^2 + E}$$

Expressing this in terms of the mean squares A, B, C we have

$$\hat{h}^2 = \frac{4\,[\lambda_1 A - \lambda_2 B + (\lambda_2 - \lambda_1)\, C]}{\lambda_1 A + (\lambda_3 - \lambda_2)\, B + (\lambda_1\lambda_3 + \lambda_2 - \lambda_1 - \lambda_3)\, C} \tag{3}$$

If $\sigma^2_{h^2}$ denotes the variance of h^2,

$$\sigma^2_{h^2} \cong \left(\frac{\partial h^2}{\partial A}\right)^2 \sigma_A{}^2 + \left(\frac{\partial h^2}{\partial B}\right)^2 \sigma_B{}^2 + \left(\frac{\partial h^2}{\partial C}\right)^2 \sigma_C{}^2$$

since A, B, C are independent of each other. Thus differentiating h^2 with respect to A, B, C respectively and substituting in the above formula, we have the estimated standard error of

$$h^2 \cong \frac{4\lambda_1\lambda_3}{P^2} [\{B + (\lambda_1 - 1)\, C\}^2\, \hat{\sigma}_A{}^2 + \{A + (\lambda_2 - 1)\, C\}^2\, \hat{\sigma}_B{}^2$$

$$+ \{(\lambda_2 - 1)\, B - (\lambda_1 - 1)\, A\}^2\, \hat{\sigma}_C{}^2]^{\frac{1}{2}} \tag{4}$$

where

$$P = \lambda_1 A + (\lambda_3 - \lambda_2)\, B + (\lambda_1\lambda_3 + \lambda_2 - \lambda_1 - \lambda_3)\, C.$$

3. Illustration

The data analysed in order to illustrate the application of these results relate to the progeny of 22 sires. The number of progeny per sire vary from six to fifty-one. The analysis of the data was carried out as indicated in Table III and the results are shown below in Table IV.

TABLE IV

Analysis of variance of first clip wool yields (in oz.) of 510 ewes, Hissar

Source of Variation	D.F.	Mean square
Between sires	21	117·50
Between dams Within sires	343	37·91
Within dams Within sires	145	28·60

For the data considered,

$$\lambda_1 = 1\cdot3891 \qquad p = 792\cdot9569$$
$$\lambda_2 = 1\cdot5167 \qquad p_1 = 51\cdot0777$$
$$\lambda_3 = 22\cdot7877 \qquad p_2 = 14708\cdot5281$$
$$p_3 = 801\cdot0540.$$

Making use of the above constants and mean squares, the estimates of components of variation and their standard errors are obtained from equation (2) and the results are presented in Table V.

TABLE V

Estimates of components of variation of first clip wool yield with their standard errors

Component of Variation	Estimate	S.E.
Additive genetic	13·8206 (35·66%)	6·8995
Dominance	12·9881 (33·51%)	14·8955
Environment	11·9486 (30·83%)	12·5382

The percentage of total phenotypic variation is given within brackets. From formulæ (3) and (4), heritability turns out to be 0·36 with an estimated standard error of 0·16.

4. SUMMARY

A knowledge of the components of variation in the character selected for, is of particular interest to the plant or animal breeder. The method of estimation of additive genetic, dominance and environmental portions of variance, from sib analysis, was described. The standard errors of these estimates and of heritability were obtained.

The data analysed relate to the first clip wool yield records of 510 progeny of twenty-two sires. The results showed 35·7% of additive genetic variation, 33·5% of dominance variation and 30·8% of environmental variation. The presence of additive genetic variation shows scope for improvement through selection.

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